ANISOTROPY OF STRENGTH IN SINGLE CRYSTALS UNDER PLANE STRAIN COMPRESSION*

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The strength of a crystal undergoing deformation depends on the crystal orientation and the geometry of deformation. In this paper, the Bishop and Hill analysis (*Phil. Mag.* **42,** pp. 414–427 and pp. 1298–1307 (1957)) is utilized to calculate the stress requirements for f.c.c. crystals of several orientations undergoing plane strain deformation. The latter is frequently found in plastic working operations such as rolling, deep drawing, and wire flattening. Compression tests were conducted on Permalloy (4% Mo–17% Fe–79% Ni) single crystals and on polycrystalline material with the samples confined to a channel to prevent lateral spreading. Strength differences by a factor of two may be obtained this way. The present findings are in good agreement with analysis, although some results are complicated by deformation banding and lattice rotations.

ANISOTROPIE DE RESISTANCE DE MONOCRISTAUX SOUMIS A UNE COMPRESSION EN ETAT PLAN DE DEFORMATION

La résistance d'un cristal soumis à une déformation dépend de l'orientation de ce cristal et de la géométrie de la déformation. Dans cet article, les auteurs utilisent l'analyse de Bishop et Hill (*Phil. Mag.* **42,** pp. 414–427 et pp. 1298–1307 (1957)) pour calculer les conditions de contraintes relatives à des cristaux c.f.c. de différentes orientations soumis à une déformation en état plan de déformation. Ce type de déformation se rencontre en effet dans de nombreuses opérations de déformation plastique, telles que le laminage et l'emboutissage. Les essais de compression ont été exécutés sur des monocristaux de Permalloy (4%Mo–17%Fe–79%Ni); ainsi que sur des échantillons polycristallins, les éprouvettes étant sollicitées à l'aide d'un équipement destiné à empêcher toute expansion latérale. On peut obtenir de cette manière des différences de résistance variant du simple au double. Les résultats des essais sont en bon accord avec l'analyse théorique, bien que certains résultats se compliquent par l'apparition de bandes de déformation et par la rotation du réseau.

DIE ANISOTROPIE DER FESTIGKEIT VON EINKRISTALLEN BEI KOMPRESSION MIT EBENER VERZERRUNG

Die Festigkeit eines plastisch verformten Kristalles hängt von der Kristallorientierung und von der Verformungsgeometrie ab. In dieser Arbeit wird mit Hilfe der Analyse von Bishop und Hill (*Phil. Mag.* **42,** S. 414–427 und S. 1298–1307 (1957)) die erforderliche Spannung für die Verformung mit ebener Verzerrung von k.f.z. Kristallen verschiedener Orientierung berechnet. Diese Verformungsart liegt häufig vor beim Walzen, Ziehen und Drahtabflachen. Kompressionsversuche wurden an Permalloy (4% Mo–17% Fe–79% Ni) Einkristallen durchgeführt, sowie an polykristallinem Material, wobei die Probe zur Vermeidung einer seitlichen Ausbreitung in einen Kanal gebettet wurde. Auf diese Weise kann man Festigkeitsunterschiede bis zu einem Faktor 2 erzielen. Die Ergebnisse sind in guter Übereinstimmung mit der Analyse, obwohl einige Beobachtungen verwickelter werden durch Bandbildung bei der Verformung und durch Gitterdrehungen.

Since plastic deformation is generally accomplished by a slip or twinning process, the stresses required to deform a crystal will vary with crystal orientation and geometry of deformation. The exploitation of crystallographic texture in strengthening materials—"texture hardening"(1)—has been the subject of several recent investigations.(1–3) Of particular interest is the work of Hosford and Backofen,(3) who applied the Bishop and Hill analysis of slip(4) in calculating the yield strength of textured sheets. The latter method is a simplication of the earlier Taylor(5) analysis and places it on a theoretically sounder basis.

To date the only direct investigation of the basic theory using single crystals has been the work of Hosford,(6) who imposed axial symmetric flow in aluminum crystals by wire drawing. He found a reasonable correlation of the drawing stress with axial orientation as predicted by theory. It appears that additional experiments based on other types of imposed flow are desirable as a further test to the theory. Accordingly, the Bishop and Hill analysis is applied to calculating the compression strength of crystals undergoing plane strain deformation. This type of deformation occurs frequently in plastic working operations such as rolling, deep drawing, and roll-flattening and flat-drawing of fine wires. The latter two processes are important in the manufacture of magnetic tapes for memory device applications. In the present experiment, the analytical results were tested with single crystals of Permalloy (4% Mo–17% Fe–79% Ni) in a specially designed compression apparatus. The results are in general agreement with theory. It is observed that for a given thickness reduction, the strength can vary by a factor of two even in these f.c.c. crystals.

A similar study has been made recently by Hosford(7)

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on aluminum crystals. He used an indentation method of approximating plane strain. Good agreement between theory and experiment was likewise obtained.

THE BISHOP AND HILL ANALYSIS

The amount of plastic work per unit volume done by a tensile (or compressive) stress σ_{xx} in the x-direction is

$$dw = \sigma_{xx} d\varepsilon_{xx}, \tag{1}$$

where $d\varepsilon_{xx}$ is the incremental strain in the x-direction. This external work may be equated with that due to shear in the active slip systems,

$$dw = \tau \Sigma d\gamma_i, \tag{2}$$

where τ is the shear stress for slip and is assumed to be equal for all slip systems, and $d\gamma_i$ is the incremental shear for the ith slip system. Equating (1) and (2) yields

$$M = \frac{\sigma_{xx}}{\tau} = \frac{\Sigma d\gamma_i}{d\varepsilon_{xx}} = \frac{dw}{\tau d\varepsilon_{xx}} \cdot \tag{3}$$

As pointed out by Hosford,[3] M is a generalized Schmid factor relating the applied stress for flow to the basic shear stress for slip. It is purely dependent on orientation and on the imposed shape change. Thus, once the latter is fixed (such as plane strain), the value M will vary with orientation alone.

To obtain the appropriate value of M, Bishop and Hill applied the principle of maximum work.* In this method it is noted that an arbitrary strain generally requires five or more independent active slip systems. Since the shear stress for slip is assumed equal for all slip systems, only a limited number of stress states is capable of activating the same shear stress on the five or more slip systems. This number is twenty-eight for cubic metals which slip on $\{111\}\langle 110\rangle$ slip systems. According to the principle of maximum work, the appropriate stress state(s) is one in which the work dw of equation (3) is a maximum.

The twenty-eight stress states are reproduced in Table 1. Since these states consists of simple combinations of only six stress terms.

$$A = \sigma_{22} - \sigma_{33},\ B = \sigma_{33} - \sigma_{11},\ C = \sigma_{11} - \sigma_{22},$$
$$F = \sigma_{23},\qquad G = \sigma_{31},\qquad H = \sigma_{12},$$

* Equation (3) may be rewritten as

$$\tau = \frac{1}{M}\frac{dw}{d\varepsilon_{xx}} = \frac{dw}{\Sigma d\gamma_i}.$$

For slip to occur, the shear stress τ must be raised to the critical value for slip. This may be done either by minimizing the amount of crystallographic shear $\Sigma d\gamma_i$ (Taylor's minimum shear principle), or by maximizing the amount of plastic work dw (Bishop and Hill's maximum work principle). Both methods give equivalent results, although the latter is generally simpler to apply.

TABLE 1. The 28 stress states of Bishop and Hill*

No.	A	B	C	F	G	H
1	1	−1	0	0	0	0
2	0	1	−1	0	0	0
3	−1	0	1	0	0	0
4	0	0	0	1	0	0
5	0	0	0	0	1	0
6	0	0	0	0	0	1
7	$\frac{1}{2}$	−1	$\frac{1}{2}$	0	$\frac{1}{2}$	0
8	$\frac{1}{2}$	−1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0
9	−1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0
10	−1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
11	$\frac{1}{2}$	$\frac{1}{2}$	−1	0	0	$\frac{1}{2}$
12	$\frac{1}{2}$	$\frac{1}{2}$	−1	0	0	$-\frac{1}{2}$
13	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$
14	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$
15	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$
16	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$
17	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
18	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$
19	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$
20	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
21	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0
22	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	0
23	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0
24	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
25	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
26	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
27	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
28	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

* From J. F. W. BISHOP, *Phil. Mag.* **44,** pp. 51–64 (1953).

when referred to cubic axes, the work dw is conveniently expanded as a sum of products of stress and strain with respect to these axes. Thus

$$\begin{aligned} dw &= \sigma_{11} d\varepsilon_{11} + \sigma_{22} d\varepsilon_{22} + \sigma_{33} d\varepsilon_{33} + 2\sigma_{23} d\varepsilon_{23} \\ &\quad + 2\sigma_{31} d\varepsilon_{31} + 2\sigma_{12} d\varepsilon_{21} \\ &= (\sigma_{11} - \sigma_{33}) d\varepsilon_{11} + (\sigma_{22} - \sigma_{33}) d\varepsilon_{22} + 2\sigma_{23} d\varepsilon_{23} \\ &\quad + 2\sigma_{31} d\varepsilon_{31} + 2\sigma_{12} d\varepsilon_{12}, \end{aligned} \tag{4}$$

by noting that $d\varepsilon_{11} + d\varepsilon_{22} + d\varepsilon_{33} = 0$. Substitution of A, B, etc. into (4) and then inserting into (3) yield

$$M = \frac{1}{\tau d\varepsilon_{xx}} [-B d\varepsilon_{11} + A d\varepsilon_{22} + 2F d\varepsilon_{23} + 2G d\varepsilon_{31} + 2H d\varepsilon_{12}]. \tag{5}$$

For the case of plane strain compression under study, let x be the compression axis and z the elongation direction, we have

$$d\varepsilon_{yy} = 0,\quad d\varepsilon_{zz} = -d\varepsilon_{xx},\quad d\varepsilon_{yz} = d\varepsilon_{zx} = d\varepsilon_{xy} = 0.\dagger \tag{6}$$

† Strictly speaking, plane strain does not require that $d\varepsilon_{zx}$ be zero. The latter is usually the case in rolling a polycrystalline sample and is thus a useful simplification in extending the analysis beyond single crystals. It will be seen later that the present test setup does not restrict $d\varepsilon_{yz}$ and $d\varepsilon_{zx}$ to zero, although the symmetry of the operating slip systems in most orientations under study automatically leads to such a zero value.